

# Bayesian Fusion of TRMM Passive and Active Measurements

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**Abstract** - a Bayesian method was adopted to combine the instantaneous measurements of the Tropical Rainfall Measuring Mission (TRMM)'s radar and radiometer ([4]). The method makes multiple estimates of the rain-rate profile using the radar reflectivities assuming various plausible values for the drop size distribution (DSD) shape parameters, then selects those parameter values which produce estimates that are most consistent with the passive observations. The resulting estimates are expressed directly in terms of the DSD parameters, thus allowing one to calculate any rain-related quantity, such as rain rate profile, precipitating liquid water profile, etc. The Bayesian approach also allows one to calculate the "error bar" associated with each estimate.

## MATHEMATICAL APPROACH

Combining the simultaneous measurements of a microwave radar and a passive radiometer observing the same event can help resolve the ambiguities inherent in single-instrument attempts at rain retrieval: indeed, the fine resolution of the radar measurements should compensate for the corresponding ambiguity in the radiometer measurements (e.g. in detecting the freezing level), while the robustness of the radiometer measurements should reduce the error which the radar can make when estimating integrated quantities (errors that are due mostly to the significant dependence of the radar backscatter on the unknown hydrometeor size).

We chose a Bayesian approach to implement such a combined algorithm in the case of TRMM, the Tropical Rainfall Measuring Mission ([5]). The advantage of such an approach is that it gives as much importance to the measurements of the radar and of the radiometer as their respective intrinsic ambiguities warrant, while avoiding all ad hoc shortcuts that might introduce large biases in the rain estimates.

Starting with the idea advocated some time ago by J. Weinman ([7]) and H. Kumagai ([3]) of estimating the "high resolution" rain profile using the spatially detailed radar reflectivities, while constraining

this estimation to be consistent with the (independent) estimate of the total attenuation derived from the passively-measured brightness temperatures, we adopt a two-step procedure: first, since the radar-rain relations depend mostly on the drop size distribution (DSD) parameters  $\vec{D}$ , we use the radar reflectivities  $Z(h)$  to perform a radar-only rain-profile estimate  $R_{\vec{D}}(h)$  as a function of height  $h$ , for every possible set of values of the DSD parameters  $\vec{D}$ . The second step consists of deriving from each radar-only profile  $R_{\vec{D}}(h)$  and from the radar-estimated freezing level  $h_{ice}$  the expected brightness temperature  $T_i(\vec{D}, h_{ice})$  at the various microwave frequencies represented by the index  $i = 1, \dots, M$ . The next two sections describe how these two steps are implemented individually. To combine the results of these steps, we try to determine the probability that the rain rates at altitudes  $h_1, \dots, h_N$  are  $R_1, \dots, R_N$ , given the measured radar reflectivities  $Z(h)$  and brightness temperatures  $T_i$ :

$$\begin{aligned} \mathcal{P}(R_1, \dots, R_N, \vec{D} | Z(h), T_1, \dots, T_M) \\ = \mathcal{P}(T_1, \dots, T_M | R_1, \dots, R_N, \vec{D}, Z(h)) \\ \cdot \mathcal{P}(R_1, \dots, R_N, \vec{D} | Z(h)) \\ \cdot \mathcal{P}(T_1, \dots, T_M | Z(h))^{-1} \end{aligned} \quad (1)$$

The last term is a constant  $C$  as far as our unknowns  $R_1, \dots, R_N, \vec{D}$  are concerned. Applying Bayes's rule again to the middle term we obtain

$$\begin{aligned} \mathcal{P}(R_1, \dots, R_N, \vec{D} | Z(h), T_1, \dots, T_M) \\ = \mathcal{P}(T_1, \dots, T_M | R_1, \dots, R_N, \vec{D}, Z(h)) \\ \cdot \mathcal{P}(R_1, \dots, R_N | \vec{D}, Z(h)) \cdot \mathcal{P}(\vec{D} | Z(h)) \cdot C, \end{aligned} \quad (2)$$

with  $C$  that constant which makes the integral of the right-hand-side with respect to  $R_1, \dots, R_N, \vec{D}$  equal to 1. If we had explicit expressions for the terms in the right-hand-side of (2), all we would need to do to obtain optimal estimates  $\hat{R}_i$  of the rain rates at the various altitudes  $h_i$  given the combined data would be to evaluate the mean of  $R_i$ . We shall write down such explicit expressions in the next two sections, then return to (2) to derive the corresponding estimate of the rain rate.

## PASSIVE MODEL

The first version of the TRMM combined algorithm ignores the higher passive frequencies, and uses for the 10.7 GHz brightness temperature the forward model

$$T_1(\vec{D}, h_{ice}) = T_A - (T_A - T_0) e^{-\alpha'(\vec{D}) \int R(h)^{\beta(\vec{D})} dh} \quad (3)$$

where  $\alpha R^\beta$  is the attenuation coefficient in dB/km corresponding to a rain-rate of  $R$  mm/hr (both of whose factors  $\alpha$  and  $\beta$  depend on the DSD parameters),  $\alpha' = c\alpha$ , and  $T_A, T_0$  and  $c$  are regression coefficients obtained from careful forward simulations ([6]) comparing the integrated attenuation at 14 GHz (the TRMM radar frequency) with the 10.7 GHz radiance within the same field of view. The integral in the exponential “is taken over the entire rain column.

By analogy with the low-frequency case, we postulate the following empirical form for the brightness temperature at an arbitrary microwave frequency  $f$ :

$$T_f(\vec{D}, h_{ice}) = T_A + (\tau - T_A + T_0) e^{-\alpha'(\vec{D}) \int R'(h) dh} - \tau e^{-\rho \alpha'(\vec{D}) \int R'(h) dh} \quad (4)$$

where the coefficients  $T_A, T_0, \tau, \alpha'$  and  $\rho$  must be determined for the given band, freezing level and drop size distribution, and where  $R'(h)$  is no longer the rain rate itself but rather an ad-hoc “attenuated” version

$$R'(h) = R(h) e^{-\gamma_f \int_h^{h_t} R(h') dh'} \quad (5)$$

In (5),  $h_t$  denotes the top of the storm, and  $\gamma_f$  is a coefficient to be determined. To determine values for the coefficients  $\gamma_f, T_A, T_0, \tau, \alpha'$  and  $\rho$  appropriate for a given frequency  $f$ , simulations with a given  $\vec{D}$  and  $h_{ice}$  are used to produce pairs  $(T_f, \int R')$ , where  $R'$  is computed with several trial values of  $\gamma > 0$ : since the problem of estimating the values of the 5 remaining parameters  $(T_A, T_0, \tau, \alpha', \rho)$  that best fit the simulated data is quite difficult, we simplify it by noting that

- $T_0$  can be approximated by the average radiance when  $\int R' \simeq 0$ ,
- $T_A$  can be approximated by the apparent asymptotic radiance when  $\int R' \rightarrow \infty$ ,
- and if  $T_m$  denotes the maximum radiance in the given population and  $R_m$  the corresponding value of  $\int R'$ , then  $\tau, \alpha'$  and  $\rho$  must satisfy

$$\frac{\tau \rho}{\tau + T_0 - T_A} = e^{(\rho-1)\alpha' R_m} \quad (6)$$

$$\tau = \frac{T_A - T_m}{1 - \rho} e^{\rho \alpha' R_m} \quad (7)$$

a system which determines  $\alpha'$  and  $\tau$  in terms of  $\rho$ .

In this fashion, each  $\rho$  between 0 and 1 determines the “best-fit” parameters completely, and the  $\rho$  producing the smallest overall residual r.m.s. error between (4) and the simulated radiances is retained. The individual r.m.s. uncertainty  $\sigma(T)$  as a function of the brightness temperature is recorded for future use.

Fig. 1 shows an example of simulated data at 36.6 GHz with a 4-km freezing level, together with the fit that was achieved using the approach above. This case was chosen specifically because it was theoretically among the most difficult to treat ( $T_A < T_0 < T_m$ ).

## RADAR MODEL

In [1], we described a bayesian approach to estimate, given  $D$  and  $Z(h)$ , the mean rain profile  $R_{\vec{D}}(h)$  (given the noise in  $Z(h)$  and given other unknown factors affecting the accuracy of the model). In fact, the extended Kalman filter which was used also produces an approximation of the variance  $\delta(R)^2$  of its estimates.

There remains to define the DSD parameters  $\vec{D}$ . In [2], we defined “normalized” versions  $D''$  and  $s''$  of the mass-weighted mean drop diameter and of the relative drop diameter variance, “normalized” in the sense that the mean-drop-diameter’s and relative-drop-diameter-variance’s empirically observed correlations with the rain-rate  $R$  and with one another were factored out to produce mutually uncorrelated parameters. Thus the DSD was assumed to have the shape of a  $\Gamma$ -distribution with parameters specified by  $\vec{D} = (R, D'', s'')$ . The first version of the TRMM combined algorithm does indeed use these parameters. They are, however, unsatisfactory, for the main reason that the  $\Gamma$ -distribution model seldom fits measured DSD samples very well. To remedy that problem, and short of using the binned drop diameters themselves as parameters (of which there would then be far too many), we have chosen to apply the Karhunen-Loève decomposition to the variables  $B_1, \dots, B_{20}$  representing the number of drops whose diameters lie in twenty contiguous intervals covering the positive real numbers ( $B_{20}$  is open-ended), and retaining the 3 eigenvariables with the largest variance, while setting the values of the remaining 17 eigenvariables equal to their sample mean (which is justified since they vary much less significantly from DSD sample to DSD sample than the first 3). A detailed description of this

procedure and the resulting parametrization and radar-only retrieval will be described in an upcoming paper.

### COMBINED MODEL

Let us write  $\mathcal{G}_\Sigma$  for the O-mean Gaussian density function with variance  $\Sigma^2$ . Returning to (2), we need to calculate the mean of that function in order to obtain our optimal estimate  $\hat{R}_i$  of the rain rate at altitude  $h_i$ . Using the results of the previous two sections, this can be approximately accomplished by the integral

$$\hat{R}_i = \int R_{\vec{D}}(h_i) \left( \prod_{i=1}^M \mathcal{G}_{\sigma(T_i)}(T_i - T_i(\vec{D}, h_{ice})) \right) \mathcal{P}(\vec{D}) d\vec{D} \quad (8)$$

Similarly, the m.s. uncertainty  $\Sigma_i^2$  in this estimate can be approximated by

$$\Sigma_i^2 = \int \delta(R_{\vec{D}}(h_i))^2 \left( \prod_{i=1}^M \mathcal{G}_{\sigma(T_i)}(T_i - T_i(\vec{D}, h_{ice})) \right) \mathcal{P}(\vec{D}) d\vec{D} = \hat{R}_i^2 \quad (9)$$

In (8) and (9), we have replaced  $\mathcal{P}(\vec{D}|Z)$  by " $\mathcal{P}(\vec{D})$ " to simplify the notation. In practice, the radar reflectivities do allow one to discriminate between stratiform and convective rain, and the a priori distribution of  $\vec{D}$  is chosen accordingly (see [2] for the case of the  $\Gamma$ -distribution model).

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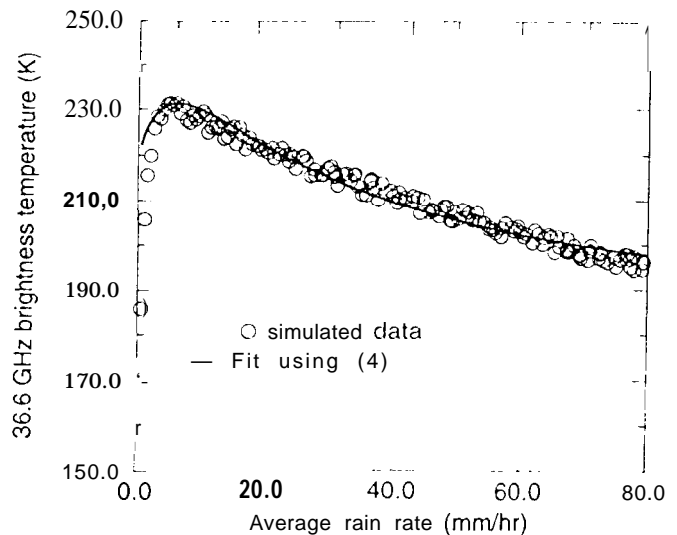


Figure 1: example of the passive model fit